SOLUTION SET VI

EXERCISE VI.1 : IMPEDANCES LOCUS

A.

First circuit We start from the admittance of the circuit portion having elements in parallel. In the expression of the admittance, only the imaginary part is a function of the frequency, hence simple to represent :

$$\underline{Y}_2 = \frac{1}{R_2} + j\omega C = \frac{1}{33} + j \cdot 12 \cdot 10^{-6} \cdot \omega$$

$$\underline{Z}_2 = \frac{R_2 \cdot \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} = \frac{R_2}{1+(\omega R_2 C)^2} - j \cdot \frac{\omega C \cdot R_2^2}{1+(\omega R_2 C)^2}$$

The impedance $\underline{Z}_2 = \frac{1}{\underline{Y}_2}$ is determined by inversion (the inverse of a line is a circle). The semicircle representing the locus of $\underline{Z}_2$ for $\omega = 0 \div \infty$ is defined by its two ends ($\omega = 0 \Rightarrow \underline{Z}_2 = R_2 = 33\ \Omega$; $\omega = \infty \Rightarrow \underline{Z}_2 = 0\ \Omega$) and its radius $\frac{R_2}{2} = 16{,}5\ \Omega$.

The impedance of the rest of the circuit is : $\underline{\underline{Z_1 = R_1 + j\omega L = 10 + j \cdot 6{,}5 \cdot 10^{-3} \cdot \omega}}$

The impedance of the whole circuit is the sum : $\underline{Z} = \underline{Z}_1 + \underline{Z}_2$:

$$\underline{\underline{Z}} = R_1 + \frac{R_2}{1+(\omega R_2 C)^2} + j\omega\left(L - \frac{C \cdot R_2^2}{1+(\omega R_2 C)^2}\right)$$

The function $\underline{Z}$ crosses the real axis in 2 points (when $\mathrm{Im}\{\underline{Z}\} = 0$) :

a. $\omega = 0$: $\underline{Z}_{(\omega=0)} = R_1 + R_2 = 10 + 33 = 43\ \Omega$

b. $L - \frac{C \cdot R_2^2}{1+(\omega R_2 C)^2} = 0 \Rightarrow \omega = \sqrt{\frac{C \cdot R_2^2 - L}{L \cdot (C \cdot R_2)^2}} = \sqrt{\frac{12 \cdot 10^{-6} \cdot 33^2 - 6{,}5 \cdot 10^{-3}}{6{,}5 \cdot 10^{-3} \cdot (12 \cdot 10^{-6} \cdot 33)^2}} = 2{,}53 \cdot 10^3$

$$\underline{Z}_{(\omega=2{,}5 \cdot 10^3)} = R_1 + \frac{R_2}{1+(\omega C R_2)^2} = 10 + \frac{33}{1+(2{,}53 \cdot 10^3 \cdot 12 \cdot 10^{-6} \cdot 33)^2} = 26{,}5\ \Omega$$

$$\omega = 2\pi f = 2{,}53 \cdot 10^3 \qquad \Rightarrow \qquad f = \frac{2{,}53 \cdot 10^3}{2\pi} = 400\ \text{Hz}$$

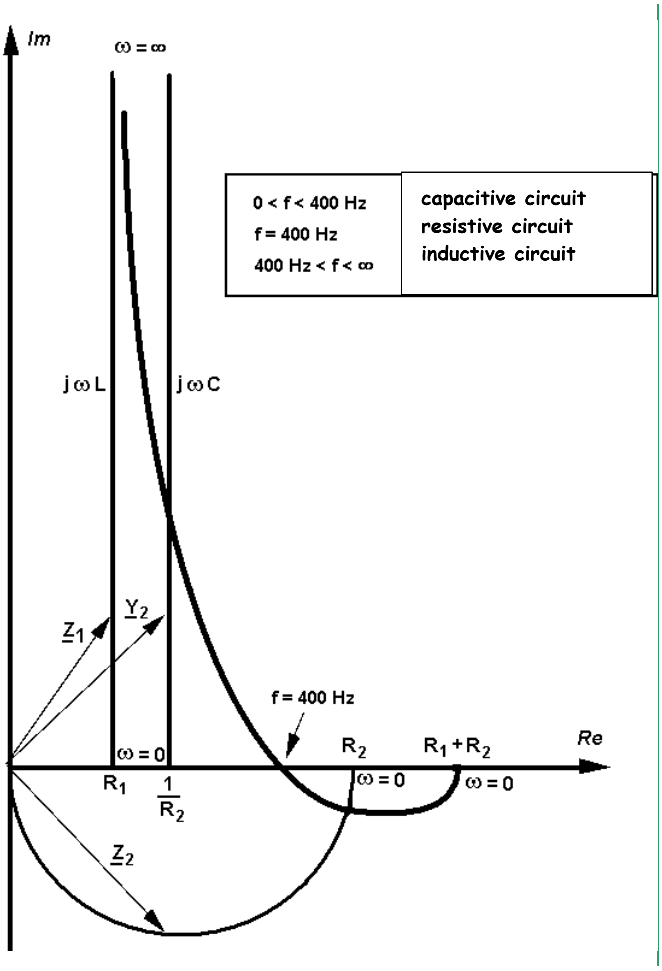
Second circuit Proceeding in the same way as for the first circuit, we find :

$$\underline{Y}_2 = \frac{1}{R_2} + \frac{1}{j\omega L} = \frac{1}{33} - j \cdot \frac{1}{6{,}5 \cdot 10^{-3}\omega}$$

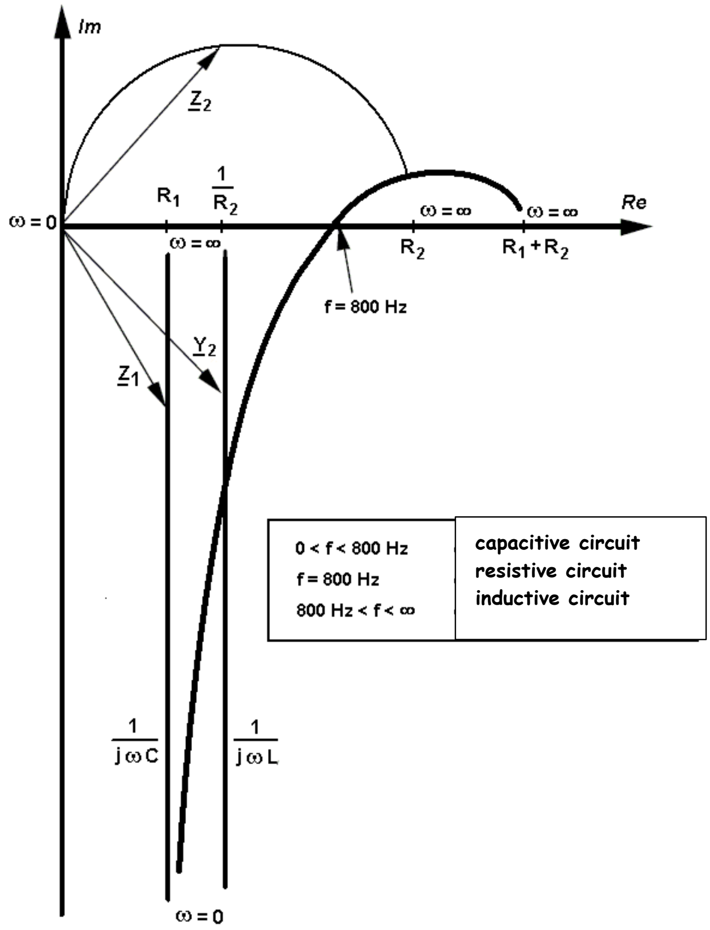
$$\underline{Z}_2 = \frac{1}{\underline{Y}_2} = \frac{R_2 \cdot (\omega L)^2}{R_2 + (\omega L)^2} + j\frac{R_2^2 \cdot \omega L}{R_2 + (\omega L)^2} = \begin{cases} 0 & \omega = 0 \\ R_2 = 33\,\Omega & \omega = \infty \end{cases}$$

$$\underline{Z}_1 = R_1 - j\frac{1}{\omega C} = \begin{cases} \infty & \omega = 0 \\ R1 = 10\,\Omega & \omega = \infty \end{cases}$$

$$\underline{Z} = \underline{Z}_1 + \underline{Z}_2 = R_1 + \frac{R_2 \cdot (\omega L)^2}{R_2 + (\omega L)^2} - j\left(\frac{1}{\omega C} - \frac{R_2^2 \cdot \omega L}{R_2 + (\omega L)^2}\right) = \begin{cases} \infty & \omega = 0 \\ R_1 + R_2 = 43\,\Omega & \omega = \infty \end{cases}$$



First circuit



Second circuit

For $\omega \to 0$, the curve of $\underline{Z}(\omega)$ tends asymptotically to the right $\mathrm{Re}\{\underline{Z}_1\} = R_1$.

B. See boxes in the graphs above.

EXERCISE VI.2 : POWER CALCULATION

A. $\underline{Z} = R + j\omega L = R + j(2\pi f)L = R + jX_L$

B. $R = \frac{U_R}{I} = \frac{60\text{ V}}{2\text{ A}} = \underline{30\ \Omega}$

$X_L = \frac{U_L}{I} = \frac{80\text{ V}}{2\text{ A}} = \underline{40\ \Omega}$

$\underline{Z} = R + jX_L = \underline{30 + j\cdot 40 = 50\cdot e^{j\cdot 53°}\ \Omega}$

C. $L = \frac{X_L}{2\pi f} = \frac{40}{100\pi} = \underline{127{,}3\text{ mH}}$

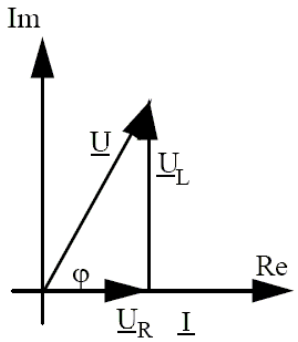
D. Rms value : $U = Z\cdot I = 50\cdot 2 = \underline{100\text{ V}}$

or : $U = \sqrt{U_R^2 + U_L^2} = \sqrt{60^2 + 80^2} = \underline{100\text{ V}}$

In complex : $\underline{U} = \underline{Z}\cdot\underline{I} = 50\cdot e^{j53°}\cdot 2\cdot e^{j0°} = \underline{100\cdot e^{j53°}\text{ V} = (60 + j80)\text{ V}}$

or : $\underline{U} = U_R + jU_L = \underline{(60 + j80)\text{ V} = 100\cdot e^{j53°}\text{ V}}$

E. Diagram of current and voltages



With $\underline{I}$ referred to 0°

Argument of $\underline{U}$: $\alpha = 53°$

Argument of $\underline{U}_R$: $\beta = 0°$

Phase difference of $\underline{I}$ and $\underline{U}$: $\varphi = \alpha - \beta = 53°$

F.

Active power

$P = R\cdot I^2 = 30\cdot 2^2 = \underline{120\text{ W}}$

or : $P = \frac{U_R^2}{R} = \frac{60^2}{30} = \underline{120\text{ W}}$

or : $P = U_R\cdot I = 60\cdot 2 = \underline{120\text{ W}}$

or : $P = U\cdot I\cdot\cos\varphi = 100\cdot 2\cdot 0{,}6 = \underline{120\text{ W}}$

Reactive power

$Q = X_L\cdot I^2 = 40\cdot 2^2 = \underline{160\text{ VAr}}$

or : $Q = \frac{U_L^2}{X_L} = \frac{80^2}{40} = \underline{160\text{ VAr}}$

or : $Q = U_L\cdot I = 80\cdot 2 = \underline{160\text{ VAr}}$

or : $Q = U\cdot I\cdot\sin\varphi = 100\cdot 2\cdot 0{,}8 = \underline{160\text{ VAr}}$

G. The compensation of the reactive powers is expressed by:

$$X_L I^2 = \frac{U^2}{X_C} = \omega C\cdot U^2 \quad\Rightarrow\quad C = \frac{X_L I^2}{\omega U^2} = \frac{160}{100\pi\cdot 100^2} = \underline{50{,}9\ \mu\text{F}}$$